Using a system’s equilibrium behavior to reduce its energy dissipation in nonequilibrium processes

Sara Tafoya,a,b,c,1,2 Steven J. Large,d,1 Shixin Liue, Carlos Bustamantea,b,c,i,j,3 and David A. Sivakd,3

*Jason L. Choy Laboratory of Single Molecule Biophysics, University of California, Berkeley, CA 94720; bHoward Hughes Medical Institute, University of California, Berkeley, CA 94720; cBiophysics Graduate Student Group, University of California, Berkeley, CA 94720; dDepartment of Physics, Simon Fraser University, Burnaby, BC V5A 1S6, Canada; eLaboratory of Nanoscale Biophysics and Biochemistry, The Rockefeller University, New York, NY 10065; fDepartment of Molecular and Cell Biology, University of California, Berkeley, CA 94720; gDepartment of Physics, University of California, Berkeley, CA 94720; hDepartment of Chemistry, University of California, Berkeley, CA 94720; iCalifornia Institute for Quantitative Biosciences, University of California, Berkeley, CA 94720; and jKavli Energy Nanoscience Institute, University of California, Berkeley, CA 94720

Edited by Attila Szabo, NIH, Bethesda, MD, and approved February 12, 2019 (received for review October 21, 2018)

Cells must operate far from equilibrium, utilizing and dissipating energy continuously to maintain their organization and to avoid stasis and death. However, they must also avoid unnecessary waste of energy. Recent studies have revealed that molecular machines are extremely efficient thermodynamically compared with their macroscopic counterparts. However, the principles governing the efficient out-of-equilibrium operation of molecular machines remain a mystery. A theoretical framework has been recently formulated in which a generalized friction coefficient quantifies the energetic efficiency in nonequilibrium processes. Moreover, it posits that, to minimize energy dissipation, external control should drive the system along the reaction coordinate with a speed inversely proportional to the square root of that friction coefficient. Here, we demonstrate the utility of this theory for designing and understanding energetically efficient nonequilibrium processes through the unfolding and folding of single DNA hairpins.

Reversible heat engines operating infinitely slowly according to the Carnot cycle do not dissipate energy: their energetic efficiency is limited only by the entropy increase of the surroundings associated with the transfer of heat from a hot to a cold reservoir. In contrast, for engines operating irreversibly, the extra nonequilibrium energy cost associated with carrying out a process at a finite rate further reduces their efficiency (1). This is the case of biological machines (2) that must operate under signaling, transport, and cell cycle time constraints. For instance, F_{1} F_{0}–ATP synthase, the primary machine responsible for ATP synthesis, can rotate up to ~350 revolutions per second (3); the bacteriophage φ29 packaging motor internalizes the 19.3-kbp viral genome into a small capsid at rates of 100 bp/s—faster than the relaxation rate of the confined DNA (4); and during sporeulation, the Bacillus subtilis DNA translocase, SpoIIIE, transfers two-thirds of its 4.2 million-base pair genome between mother cell and prespore in only 15 min (i.e., at a transfer rate of nearly 4,000 bp/s) (5). The finite time operations of these machines necessarily involve energy dissipation—often in the form of extra work—and it is of great interest to understand how they attain their large (over 70%) energetic efficiencies (6, 7).

Recently, a generalized friction coefficient—which can be obtained from equilibrium measurements—was shown to be the parameter that governs the near-equilibrium energy dissipation during a finite rate process (8). Here, we demonstrate experimentally the utility of this theoretical framework for designing energetically efficient nonequilibrium processes and propose that similar operation protocols may underlie the high efficiency observed in molecular machines. To this end, we subject single DNA hairpins to mechanical unfolding and refolding using protocols dictated by this theory; we show that these protocols systematically and significantly reduce energy dissipation during the process. DNA hairpins are ideally suited for this test, as the magnitude of the friction coefficient can be tuned by changing the molecule’s length, the free energy difference, the free energy barrier, and the transition rates between its folded and unfolded states (9).

According to this near-equilibrium linear response theory, the excess power dissipated by a system taken from an initial to a final state by varying a control parameter λ according to a protocol (time schedule) Λ is proportional to a generalized friction coefficient ζ(8):

\[ \langle P_{ex}(t) \rangle \approx \zeta(\lambda) \left( \frac{d\lambda}{dt} \right)^2. \]

\[ \zeta(\lambda) = \frac{1}{\pi} \int_{0}^{\infty} \frac{\delta F(0) \delta F(t)}{\delta F(t)} dt, \]

\[ \beta = C_{18} \]

which can be decomposed into

Significance

Biomolecular machines implement many vital activities in cells and must operate quickly and in functional directions, requiring energy dissipation. Recent experiments reveal that some evolved machines are quite energetically efficient, engendering interest in design principles that achieve such high efficiency. Recent theory predicts how to use equilibrium measurements (in the absence of driving) to design ways to drive a system to minimize energy dissipation. Here, we experimentally demonstrate the utility of this theory for designing efficient driving processes ("protocols") by rapidly unfolding and folding single DNA hairpins. We show that such designed protocols systematically and significantly reduce energy dissipation over large variations of driving speed and DNA hairpin friction. Similar protocols may underlie the high efficiency observed in molecular machines.

Author contributions: S.T., S.I.J.L., S.L., C.B., and D.A.S. designed research; S.T. performed research; S.I.J.L. and D.A.S. analyzed data; and S.T., S.I.J.L., S.L., C.B., and D.A.S. wrote the paper.

The authors declare no conflict of interest.

This article is a PNAS Direct Submission.

Published under the PNAS license.

1S.T. and S.I.J.L. contributed equally to this work.

2Present address: Scientific Application Development Group, Lumicks, Cambridge, MA 02139.

3To whom correspondence should be addressed. Email: carlosl@berkeley.edu or divakar@ucf.edu.

This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1817781116/-/DCSupplemental.

Published online March 13, 2019.
\[ \zeta(\lambda) = \beta \langle \delta F^2 \rangle_{\tau_{\text{relax}}} \]  

the product of the force variance \( \langle \delta F^2 \rangle \) and the force relaxation time

\[ \tau_{\text{relax}}(\lambda) = \int_0^\infty \langle \delta F(0) \delta F(t) \rangle_{\lambda} \, dt. \]  

Here, \( \langle \ldots \rangle \) denotes a nonequilibrium average over system response to a given protocol, whereas \( \langle \ldots \rangle_{\lambda} \) denotes an equilibrium average over system fluctuations at fixed control parameter \( \lambda \). Notice that this framework makes no assumptions about the hairpin dynamics being Markovian and that the force autocorrelation function is well defined regardless of whether the dynamics is Markovian (11).

It can be shown (12) that, near equilibrium, the driving protocol that minimizes the dissipation for a given total duration, \( \lambda(t)_{\text{designed}} \), must proceed with a velocity proportional to the inverse square root of the local friction coefficient \( \zeta(\lambda) \), \( d\lambda(t)_{\text{designed}} / dt \propto \zeta(\lambda)^{-1/2} \). The proportionality is fixed by the total duration of the protocol, and therefore, changing it corresponds to a global rescaling of all velocities. Other approaches to minimizing work (13, 14) require detailed knowledge of both the system equilibrium landscape and nonequilibrium dynamics and thus, are experimentally challenging.

To obtain the generalized friction coefficient of the DNA hairpin, we monitored the equilibrium force fluctuations of molecules tethered between two optical traps at various fixed trap separations, \( X \). For very small or very large trap separations, the force fluctuates around a single mean value corresponding to the folded or unfolded conformation, respectively (Fig. 1A); for intermediate trap separations, the force fluctuates between two different values, reflecting the hopping dynamics of the DNA hairpin sampling the folded and unfolded conformations (Fig. 1A).

For each separation \( X \), we calculated the force autocorrelation function, \( \langle \delta F(0) \delta F(t) \rangle_X \) (Fig. 1C); as expected, in the hopping regime, the force variance is larger, and fluctuations decay more slowly than when an extreme trap separation holds the DNA hairpin in a single conformation.

Next, we calculated the force variance (Fig. 1D) and the force relaxation time (Fig. 1E) from the force autocorrelation function. The force variance peaks at an intermediate trap separation, \( X_{1/2} \), where the hairpin spends roughly equal time between the folded and unfolded conformations. Likewise, the force relaxation time peaks at \( X_{1/2} \), reflecting that, to equilibrate, the hairpin must relax across the barrier separating the folded and unfolded states. At room temperature, a 1-μm bead experiencing Stokes drag (with friction \( \gamma = 3\pi R \eta \) for water viscosity \( \eta \) and bead radius \( R \)) in water and confined by a \( k = 0.25\text{-pN/nm} \) optical trap has a relaxation time \( \gamma/k \sim 30 \mu s \) (15), orders of magnitude below the minimum observed relaxation time of the entire construct—indicating that the beads do not significantly impact the relaxation times observed in the experiments, typically in the low millisecond range. Moreover, the generalized friction coefficient—the product of force variance and force relaxation time (Eq. 3)—also peaks at \( X_{1/2} \) (Fig. 1F).

As mentioned above, the theory predicts that (near equilibrium) the minimum dissipation protocol proceeds with a pulling speed—or velocity of the steering trap—that scales as the inverse square root of the friction coefficient (8): pulling fast at extreme separations, where the friction coefficient is small, and slow around \( X_{1/2} \), where friction peaks. Intuitively, a slow velocity near \( X_{1/2} \) provides more time for thermal fluctuations to induce the unfolding or folding of the DNA hairpin without additional work input and therefore, decreases the work required to drive the DNA hairpin between conformations (12). To ease its implementation, the designed protocol that minimizes dissipation was approximated by a trap velocity profile with a simple piecewise-constant acceleration (Fig. 1G). The resulting designed protocols (Fig. 1H) differ substantially from naive protocols that proceed at constant velocity and that are completed in the same elapsed time. In particular, instantaneous driving velocities varied by a factor of approximately six within a given designed protocol.

Next, we measured force as a function of trap separation during designed and naive protocols with total durations ranging from 3.7 to 0.13 s. These force separation curves of naive and designed protocols display significant differences in the force at which the DNA hairpins unfold/refold (Fig. 2A). Fig. 2B shows the distributions of unfolding force differences, \( F_{\text{designed}}^U - F_{\text{naive}}^U \).
and refolding force differences, $F_R^{\text{naive}} - F_R^{\text{designed}}$, obtained for three different protocol durations. As predicted by theory, on average, the DNA hairpin unfolded at lower forces and refolded at higher forces during the designed protocols than during the naive protocols, and the magnitude of the mean force difference is greater for faster protocols (Fig. 2C). These results imply that the designed protocols display lower hysteresis than naive, a trend that is more prominent in faster protocols where the system is driven farther from equilibrium.

Analogous to Fig. 2A, Fig. 3A depicts the cycle work for a typical realization of an unfolding/refolding cycle. According to Eq. 1, when driving a system at a constant velocity, more work is dissipated at trap separations where the friction coefficient is larger. Consistently, the constant velocity protocols produce higher dissipation around $X_{1/2}$ for all durations (Fig. 3B); by contrast, designed protocols show a substantially flatter dissipation profile across different trap separations (Fig. 3B), and overall, they induce consistently less dissipation during an unfolding–refolding cycle than naive protocols (Fig. 3 C and D). Within the linear response regime, the mean dissipated work equals one-half the work variance; SI Appendix, Fig. S3 shows that, in our experiments, this relation holds for slower protocols.

The data presented here correspond to a DNA hairpin that allowed relatively rapid folded–unfolded equilibration such that transitions to the folded or unfolded conformations occurred even for 0.13-s protocols. This feature allowed us to interrogate the hairpin’s nonequilibrium response over a broad range of protocol durations. In SI Appendix, we show that these results also hold for a different DNA hairpin sequence with significantly (~100 times) slower equilibration.

In summary, we have sampled the equilibrium force fluctuations in DNA hairpins, displaying the dynamics of a two-state system (Fig. 1). We showed that the generalized friction coefficient—determined from such equilibrium fluctuations—can be used to design driving schedules (Fig. 1G) that significantly improve the overall performance of molecular motors.
reduce the excess work dissipated compared with constant velocity schedules (naive protocols) completed in the same total time (Fig. 3D). This result held for protocol durations that vary by a factor of ≈30 (Fig. 3D), even when driven far from equilibrium (dissipating up to ≈10 kBT, which greatly exceeds the ≈1–kBT energy fluctuations at equilibrium). These observations indicate that this near-equilibrium theory is still able to reduce dissipation even beyond the regime of the theory’s strict validity.

This experiment represents the design and implementation of a single-molecule protocol that systematically reduces the nonequilibrium energy dissipation in a process constrained to a finite duration.

These results have immediate applications in the streamlining of single-molecule experiments and steered molecular dynamics simulations (17). For instance, when using the Jarzynski equality or Crooks fluctuation theorem to infer the free energy difference in a given process (such as protein unfolding), the farther the system is from equilibrium during experiment or simulation, the slower the rate of convergence and accuracy of the free energy estimator, which depends inversely on the energy dissipated (18). Therefore, by sampling the equilibrium fluctuations of a biomolecular process, it should be possible to estimate the generalized friction coefficient across the control parameter landscape; next, it would be possible to craft nonequilibrium protocols that dissipate significantly less energy, thereby speeding up the convergence and increasing the accuracy of any given free energy estimator.

There are tantalizing hints of molecular machines conserving energy while operating out of equilibrium (4, 19): the ϕ29 DNA packaging motor is more likely to slow down and pause at high packaging fractions, where the storing of additional DNA involves significantly higher dissipation, and translating ribosomes facing RNA hairpins—that impose a large barrier to translation—change “gear,” operating slower while crossing the barrier (20). Based on the theoretical framework presented here, both cases can be seen as examples in which the molecular machines implement driving protocols that proceed slower where the friction coefficient is higher, thereby reducing dissipation and increasing their efficiency. We hypothesize that a molecular biophysical system can waste less energy through naturally evolved dynamics that is rationalizable in terms of the generalized friction coefficient; specifically, such molecular motors may have evolved to slow down their operation in regions of their control parameter space corresponding to high values of the friction coefficient as a way to harness fluctuations from the thermal bath, thus improving their operation efficiency.

The agreement of theory (8) and our experiments suggests extensions to more complex contexts. In particular, we conjecture that molecular machines may have evolved to slow down in regions of large friction and speed up in regions of small friction. The rotary motor F1-ATP synthase is known to be a remarkably efficient machine (21), where the F$_0$ subunit—powered by proton flow down a concentration gradient—forces rotation of the γ-subunit, a molecular crankshaft that drives synthesis of ATP by F$_1$ (6). After attaching a magnetic bead to the crankshaft of F$_1$ (22), one could—allogous to the procedure described in this study—use a magnetic tweezers instrument to hold the bead at various angles so as to extract the equilibrium torque fluctuations of the rotary crankshaft, from which one could extract the friction coefficient at each position (in this experiment, the angle corresponds to the control parameter for driving F$_1$ in analogy to the trap separation for driving the unfolding of the DNA hairpin). One could then estimate the minimum dissipation protocol and determine the ratio of energy input (work done to rotate the crankshaft) to energy output (ATP molecules synthesized) (22, 23) for designed and naive protocols. These ratios quantify the energetic efficiency with which the respective protocols induce F$_1$ to synthesize ATP, and their difference determines the energetic savings.

We have seen here that the linear response theory provides a useful qualitative guide to design protocols that systematically require less work than naive ones. Moreover, since this theoretical framework naturally generalizes to stochastic protocols (24), future experiments could be designed to more closely match autonomous machines driven by fluctuating forces. Insights from the experiments designed with this framework should provide a deeper understanding of the nonequilibrium energetic efficiency of biomolecular machines and ultimately, guide the operation of efficient synthetic nanomachines.

Materials and Methods

Basic Optical Trap Setup. High-resolution force separation measurements were conducted on a dual-trap instrument using a solid-state 1,064-nm laser as described previously (25). Traps were calibrated as previously described (26). DNA tethers were formed between a 0.90-μm-diameter streptavidin-coated bead and a 1-μm-diameter—very slightly smaller (Sigma-Aldrich) coated bead (Calbiochem) included in the buffer to prevent the formation of reactive singlet oxygen, thus increasing the lifetime of the DNA tethers.

DNA Molecules. Hairpin DNA sequence instructions were selected to display hopping dynamics such that detecting X$_{1;2}$ was accessible experimentally—very fast hopping dynamics were difficult to distinguish from noise, and very slow dynamics required long periods of data acquisition and laser exposure before pulling experiments. Minimizing laser exposure avoids molecule photodamage. All data in SI Appendix are from sequence 1, GAGTCCTGACCTGTTCCTTTTTTTTCCCCGTGATAGGTCA, which was previously characterized and exhibited appropriate hopping dynamics (τ$_{eq}$ ≈ 0.24 s) (9). All data in the text are from sequence 2, TACCTGATGAGCTCTTCTTTCCTTTTTCACCGGTAGCTGTA, the result of modifying sequence 1 to increase GC content at the loop neck. This change in sequence is expected to facilitate nucleation of the native conformation and to avoid molecule misfolding (27).

Bead size variation, small differences in chemical attachments, and non-specific interactions with the bead surface can lead to molecule to molecule variation. We minimized the contribution of trap distance variation by subtracting the value of X$_{2;3}$ from more than 20 friction estimates, the farthest the system is from equilibrium (dissipating up to ≈30 (Fig. 3D) ). This result held for protocol durations that vary between the two traps. This error was introduced as a small difference of a few millivolts between the instruction given by the computer and the actual voltage between the two traps. This error was introduced as a systematic error of $\sim$1% (∼0.1 mV) measured independently in each of 20 different molecules. From these measurements, the linear response of the trap separation to the hopping regime. Changes in separation are instantaneous but are limited by the response of the mirror controlling the steering mirror of the trap (≈2 ms).

Equilibrium force fluctuations at each of several fixed separations were measured independently in each of 20 different molecules. From these fluctuations, the generalized friction coefficient was estimated using Eq. 2. At each separation, we jackknife resampled from each of 20 force estimation to calculate the mean generalized friction and SE (29).

We fit several piecewise-constant acceleration profiles of protocol velocity to the minimum dissipation one (d$/delta_n$/dt = $\alpha \cdot |x(t)|^{-1}$ predicted from the empirically determined generalized friction $\zeta$). Each model velocity profile has constant (zero acceleration) far away from X$_{1;2}$ and in the immediate vicinity of X$_{1;2}$. Constant acceleration regions interpolate...
between these constant velocity regions. The model parameters are the region boundaries and the constant velocities. Different model velocity profiles impose different symmetries, such as inversion symmetry about $X_i$, thus reducing the number of free parameters. We used the velocity profile (Fig. 1) that minimized the Akaike Information Criterion (30), a measure of a model's balance between accuracy and complexity.

**Naïve and Designed Protocols.** We estimate the work $W$ during a trajectory of forces $F_i$ and separations $X_i$ at $n$ discrete time points by numerical integration:

$$W = \sum_{i=2}^{n-1} \frac{F_i + F_{i+1}}{2} (X_{i} - X_{i-1}).$$

There are $14, 9, 8, 8, 10, \text{ and } 9$ separate molecules sampled with $888 (444), 590 (295), 396 (198), 590 (295), 592 (296), \text{ and } 472 (236)$ individual realizations (full profiles impose different symmetries, such as inversion symmetry about $X_i$, respectively. The cycle work (hysteresis) cycles of protocols for durations of $0.13, 0.24, 0.48, 0.93, 1.8, \text{ and } 3.7$ s, respectively. The cycle work (hysteresis) $W_{\text{cycle}} = W_{+} - W_{-}$ for each protocol duration, we calculate the work along naive and designed protocols for a given molecule instead of raw unfolding/refolding forces.

The excess power in a protocol interval (Fig. 3B) is calculated by adding the total unfolding work in an interval $\Delta X$ to the total refolding work in the same interval and dividing by the time taken for the protocol to traverse that separation interval. Finally, the power in each interval is normalized by the average naive excess power (averaged over the entire protocol).

**ACKNOWLEDGMENTS.** We thank Nancy Forde, John Bechhoefer, Aidan Brown, and Kamdin Mirsanaye (Simon Fraser University); Michael Woodside (University of Alberta); and Ronen Gabilon and Antony Lee (University of California, Berkeley) for useful discussions. This work is supported in part by the Nanomachines program (KC1203) funded by the office of Basic Energy Sciences of the US Department of Energy [contract no. DE-AC02-05CH11231 (to C.B.)]; the University of California Mexus graduate fellowship (to S.T.); Natural Sciences and Engineering Research Council of Canada (NSERC) Canada Graduate Scholarships-Master’s and Alexander Graham Bell Canada Graduate Scholarships-Doctoral (to S.J.L.); the Howard Hughes Medical Institute (C.B.); an NSERC Discovery Grant (to D.A.S.); the Faculty of Science, Simon Fraser University through President’s Research Startup Grant (to D.A.S.); and a Tier-II Canada Research Chair (D.A.S.).